A NUMERICAL ALGORITHM FOR THE TELEGRAPH INVOLUTORY PROBLEM WITH NEUMANN CONDITIONS

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Introduction. Delay differential equations are universal phenomena applied their models in engineering systems to behave like a real process [1-5].

In general, for the solutions of delay differential equations, we need to give the values of unknown functions on some segments. Initial conditions in one point are not enough to get the solution of delay differential equations. For the first time, in an experiment measuring the population growth of a species of water fleas, Nisbet [7] tried to use delay differential equations with reversal time. He reversed time to get the solution of functional differential equations with a given value of the unknown function on one point. Such types of functional differential equations are called involutory differential equations. The time reversal problem is a special case of involutory problems.

We obtained an equivalent initial value problem for the fourth-order ordinary differential equations to the initial value problem for second-order linear differential equations with damping term and involution.

The theorem on stability estimates for the solution of the initial value problem for the second-order ordinary linear differential equation with damping term and involution was proved. Theorem on the existence and uniqueness of the bounded solution of initial value problem for second-order ordinary nonlinear differential equation with damping term and involution was established.

Involutory telegraph partial differential equations are not investigated before. The present paper is devoted to studying involutory telegraph equations. Fourier series, Laplace, and Fourier transforms can be applied to the solutions of involutory telegraph problems. When coefficients of the space operator are dependent on time and space variables, these methods are not applicable. In the present paper, the first and second-order accuracy difference schemes for the numerical solution of the initial boundary value problem for one-dimensional telegraph equations are constructed. Some numerical results are explained.

Numerical algorithm for the solution of the telegraph involutory partial differential equation. We present the algorithm for the numerical solution of the initial boundary value problem

$$\begin{cases} \dfrac{\partial^2 u(t,x)}{\partial t^2} + \dfrac{\partial u(t,x)}{\partial t} - u_{xx}(t,x) - b u_{xx}(-t,x) \\ = (\cos(t) - b\sin(t))\cos(x), x \in (0,\pi), -\pi < t < \pi, \\ u(0,x) = 0, u_t(0,x) = \cos(x), x \in [0,\pi], \\ u_x(t,0) = u_x(t,\pi) = 0, t \in [-\pi,\pi] \end{cases} \quad (1)$$

for the one-dimensional telegraph type involutory partial differential equation with the Neumann condition. The exact solution problem (1) is $u(t,x) = \sin(t)\cos(x)$, in $0 \le x \le \pi, -\pi \le t \le \pi$. For the approximate solutions of the problem (1), using the set of grid points

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$$[-\pi,\pi]_\tau \times [0,\pi]_h = \{(t_k,x_n): t_k = k\tau, -N \le k \le N, N\tau = \pi, x_n = nh, 0 \le n \le M, Mh = \pi\},$$

we get the first order of accuracy in t difference scheme

$$\begin{cases} \dfrac{u_n^{k+1} - 2u_n^k + u_n^{k-1}}{\tau^2} + \dfrac{u_n^{k+1} - u_n^k}{\tau} \\ -\dfrac{u_{n+1}^{k+1} - 2u_n^{k+1} + u_{n-1}^{k-1}}{h^2} - b\dfrac{u_{n+1}^{-k+1} - 2u_n^{-k-1} + u_{n-1}^{-k-1}}{h^2} \\ = (\cos(t_{k+t}) - b\sin(t_{k+1}))\cos(x_n), \\ -N+1 \le k \le N-1, 1 \le n \le M-1, \\ u_n^0 = 0, \dfrac{u_n^1 - u_n^0}{\tau} = \cos(x_n), 0 \le n \le M, \\ u_1^k = u_0^k = 0, u_M^k = u_{M-1}^k = 0, -N \le k \le N \end{cases} \tag{2}$$

and second order of accuracy in t difference scheme

$$\begin{cases} \dfrac{u_n^{k+1} - 2u_n^k + u_n^{k-1}}{\tau^2} + \dfrac{u_n^{k+1} - u_n^k}{2\tau} \\ -\dfrac{u_{n+1}^k - 2u_n^k + u_{n-1}^k}{2h^2} - \dfrac{u_{n+1}^{k+1} - 2u_n^{k+1} + u_{n-1}^{k+1}}{4h^2} \\ -\dfrac{u_{n+1}^{k-1} - 2u_n^{k-1} + u_{n-1}^{k-1}}{4h^2} - b\dfrac{u_{n+1}^{-k} - 2u_n^{-k} + u_{n-1}^{-k}}{2h^2} \\ -b\dfrac{u_{n+1}^{-k+1} - 2u_n^{-k+1} + u_{n-1}^{-k+1}}{4h^2} - b\dfrac{u_{n+1}^{-k-1} - 2u_n^{-k-1} + u_{n-1}^{-k-1}}{4h^2} \\ = (\cos(t_k) - b\sin(t_k))\cos(x_n), \\ -N+1 \le k \le N-1, 1 \le n \le M-1, \\ u_n^0 = 0, \dfrac{-u_n^2 + 4u_n^1 - 3u_n^0}{2\tau} = \cos(x_n), 0 \le n \le M, \\ -u_2^k + 4u_1^k - 3u_0^k = 0, -3u_M^k + 4u_{M-1}^k - u_{M-2}^k = 0, -N \le k \le N. \end{cases} \tag{3}$$

They are systems of algebraic equations and they can be written in the matrix form

$$\begin{gathered} u_{n-1} + Bu_n + Cu_{n+1} = D\varphi_n, 1 \le n \le M-1, \\ 3u_0 = 4u_1 - u_2, 3u_M = 4u_{M-1} - u_{M-2}, \end{gathered} \tag{4}$$

where A, B, C are $(2N + 1) \times (2N + 1)$ matrices and $D = I_{2N+1}$ is the identity matrix, φ_n and u_s are $(2N + 1) \times 1$ column vectors. For the approximate solutions of the problem (1). We will apply the modified Gauss elimination method to solve the matrix equation by following the form

$$u_n = \alpha_{n+1}u_{n+1} + \beta_{n+1}, n = M-1, \dots, 1, \tag{5}$$

where $u_M = (I - \alpha_M)^{-1}\beta_M, \alpha_j$ $(j = 1, \dots, M-1)$ are $(2N + 1) \times (2N + 1)$ square matrices, β_j $(j = 1, \dots, M-1)$ are $(2N + 1) \times 1$ column matrices, $\alpha_1 = I$, $\beta_1$is zero matrices and

$$\begin{cases} \alpha_{n+1} = -(B + C\alpha_n)^{-1}A, \\ \beta_{n+1} = (B + C\alpha_n)^{-1}(D\varphi_n + C\beta_n), n = 1, \dots, M-1. \end{cases}$$

Numerical analysis. The numerical solutions are recorded for different values of N and M, and u_n^krepresents the numerical solution of this difference scheme $u(t_k, x_n)$. Table 1 is constructed for $N = M = 40; 80; 160$ respectively and the errors are computed by

$$E_M^N = \max_{-N \leq k \leq N, 1 \leq n \leq M-1} |u(t_k, x_n) - u_n^k|.$$

If N and M are doubled, the values of the errors are decreasing by a factor of approximately $1/2$ for the first order difference scheme (2) and $1/4$ for the second order of accuracy scheme (3). The errors presented in the table 1 indicates the accuracy of difference scheme. We conclude that, the accuracy increases with the second order approximation.

Table 1. Error analysis E_M^N

Difference schemes/N=M	40	80	160
(2)	0.1013	0.0496	0.0200
(3)	0.0080	0.0020	5.0452e-04

Conclusion. In the present paper, telegraph-type involutory partial differential equations are studied. The first and second-order accuracy difference schemes for the numerical solution of the initial boundary value problem for one-dimensional telegraph type involutory partial differential equations are constructed. Numerical analysis and discussions are presented.

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